ACCELERATED FATIGUE TESTS FOR AUTOMOTIVE CHASSIS PARTS DESIGN: AN OVERVIEW

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Abstract *Fatigue strength is of prior concern for automotive chassis parts design. In a reliability framework, its assessment still requires long and expensive test campaigns. In particular, high-cycle endurance tests usually involve several specimens undergoing million-cycle procedures.*

In order to reduce the specimen number (e.g. expensive prototypes) and to shorten testing procedures (i.e. time-to-result delay), Accelerated Life Tests (ALT) are developed under the constraint of the needed high-reliability assessment.

This paper reviews a selected fatigue test procedure currently used in the automotive industry (i.e. StairCase), then addresses a comparison of the standard estimation method (i.e. Dixon & Mood) and the ALT application. Thus, numerical simulations are used in order to assess the reliability of these test procedures and to compare their performance. Finally, the application of ALT for chassis parts is introduced and discussed.

1 INTRODUCTION

Automotive chassis parts are usually designed with respect to multiple requirements, such as stiffness, noise and vibrations, mechanical strength under static and dynamic loads, wear, etc. Among all these requirements, reliability under cyclic loads is of prior concern. Actually most of the failure modes are characterized by the high cycle fatigue phenomena. The aim of this framework is to optimize fatigue test plans with high reliability assessment.

Fatigue test procedures involve cyclic stresses applied during a given number of cycles or up to failure; their objective is to guarantee a risk of failure lower than a threshold (e.g. failure probability of 10^{-6}) which can be related to fatigue strength during a reference lifespan (e.g. 10^6 load cycles).

A probabilistic approach such as the Stress-Strength method [1, 2, 3 and 4], takes into account the stress distribution and an acceptance risk leading to set a required strength distribution, which is used as a target for the fatigue test result.

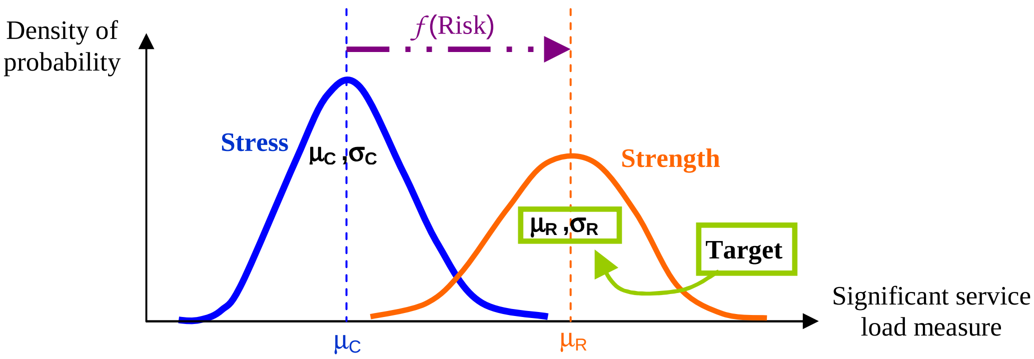


Figure 1: Stress-Strength method – target characteristic of the strength (μ_R ,σ_R), as a result of the stress distribution (μ_C,σ_C) and an acceptance risk

The testing methods in use at PSA PEUGEOT CITROEN (e.g. StairCase, Locati and StairCase-Locati [5]) give an experimental measure of the strength distribution leading to an average called m_R, and a standard deviation called s_R; these parameters have to be compared with the required values μ_R and σ_R, respectively. At present, these procedures involve some shortcuts:

- Series of test runs can last up to 3 months. As development and qualification time schedules are hardly reduced, procedure robustness should be checked as some hypotheses are not respected anymore.
- The number of test runs at PSA and its suppliers due to fatigue tests is very large (at least 10^8 cycles / year), so that their reduction would imply significant cutting of costs and delays.
- Finally, the procedures have not been updated for the last 15 years, while academic researches on accelerate testing methods have been pursued with success, in particular for the electronic industry [6].

Hence, in this paper we first focus on one PSA fatigue testing method (i.e. StairCase procedure along with a standard result analysis provided by Dixon & Mood) (§.2). Then we consider an alternative result analysis based on Accelerated Life Testing Model applied to StairCase results (§.3). Their comparison is discussed by numerical simulation. Discussions and perspectives end (§4).

2 TESTING METHOD USED AT PSA

2.1 StairCase (SC) and Dixon&Mood (D&M) methods

The StairCase method [7] is a fatigue test procedure used to estimate the fatigue strength for a sampling of parts. The method is based on an iterative principle, several parts are successively tested. The test on the current part depends on the result of the previous one, as shown on the following scheme:

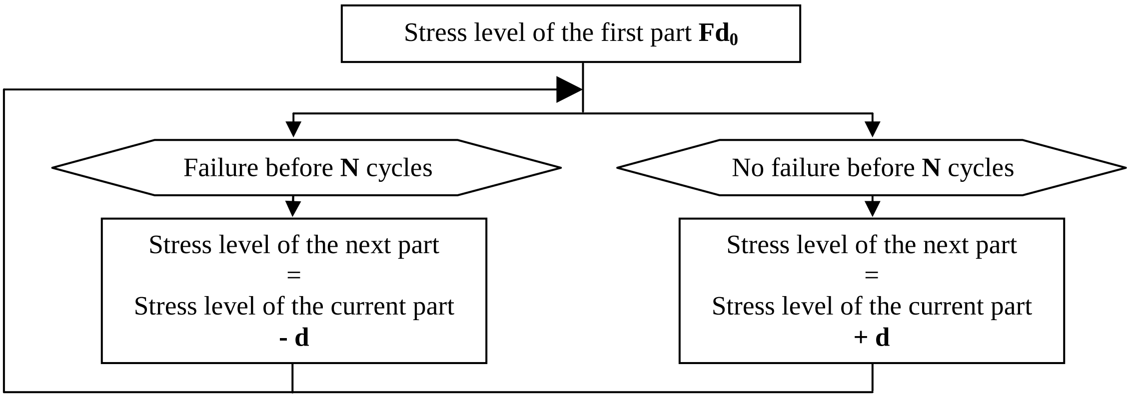


Figure 2: StairCase method algorithm

where:
- **Fd_0** is the first stress level for the first part of the sample. At the beginning of the procedure, **Fd_0** is set as close as possible to the predicted fatigue strength mean.
- **d** is the stress step between two consecutive levels. At the beginning of the procedure, **d** is set as close as possible to the predicted fatigue strength standard deviation.
- **N** is the number of cycles to censure, it is set at 10^6 cycles.

In the literature there are many ways to estimate the fatigue strength with the StairCase results [8, 9]; at PSA, the Dixon & Mood (D&M) method [7] is applied. Thus, the mean and the standard deviation of the fatigue strength are calculated as follows:

$$m = \left[F_0 + d.\left(\frac{A}{Ne} \pm \frac{1}{2} \right) \right] \quad ; \quad s = 1{,}62.d.\left(\frac{Ne.B - A^2}{Ne^2} + 0{,}029 \right) \tag{1,2}$$

where the latter is applied only if:

$$\frac{Ne.B - A^2}{Ne^2} > 0{,}3 \tag{3}$$

with:
- **F_0** the lowest stress level used in the test
- **N_e** total number of the least frequent event (specimen failed or not) for the overall test
- **$A = \Sigma\, i n_i$** used in the average calculation
- **$B = \Sigma\, i^2 n_i$** used in the standard deviation calculation
- **i** integer counter identifying the stress level, the **0** level corresponds to **F_0**
- **n_i** occurrence of the least frequent event at the **i** level
- **- 1/2** if the failure is the **least** frequent event
- **+ 1/2** if the failure is the **most** frequent event

Please note that PSA usually needs for 7 parts in order to estimate the fatigue strength mean, whereas the standard deviation of the fatigue strength is actually not determined with this testing method.

2.2 Numerical Simulation of the SC with D&M Analysis

In order to assess the efficiency of the SC method, numerical simulations are built up: this approach provides a comparing tool with respect to the quality of the estimation (i.e. convergence and bias) taking into account the cost (e.g. number of parts) and the time (e.g. number of cycles) needed for every test plan. Moreover, we will check if some parameters or hypothesis affect the final result in an unreliable way.

We generate samples, i.e. tests results, based on:
1. a normal distribution,
2. a theoretical fatigue strength distribution (μ,σ),
3. a number of parts (considering 7 up to 1000 specimens).

The computer simulation is designed to analyze 1000 runs of the procedure, giving 1000 results of each sample size in order to provide a distribution of calculated averages and standard deviations.

Here, these simulations are made according to "ideal" test conditions in order to check the convergence: this means a starting stress level, Fd_0 equal to the real fatigue strength average, μ and a step stress, d equal to the real fatigue strength standard deviation, σ.

Let us consider a strength (i.e. Normal distribution [μ=100 ; σ=8]). The simulation drives a SC protocol, applying the D&M approach, which leads to the calculated fatigue strength distribution parameters: figure 3 shows the distribution of the mean and the standard deviation estimations obtained for 1000 StairCase simulations for a 1000 sample size.

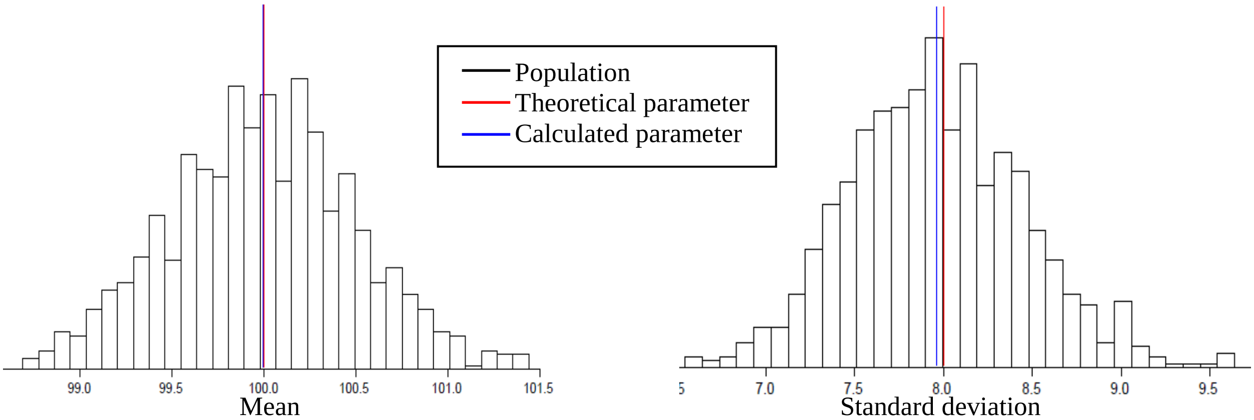


Figure 3: Distribution parameters of 1000 runs of a 1000 sample size StairCase simulation.

We can assess the convergence of the method by plotting the mean of each parameter estimates (mean and standard deviation) as a function of the sample size (figures 4 and 5).

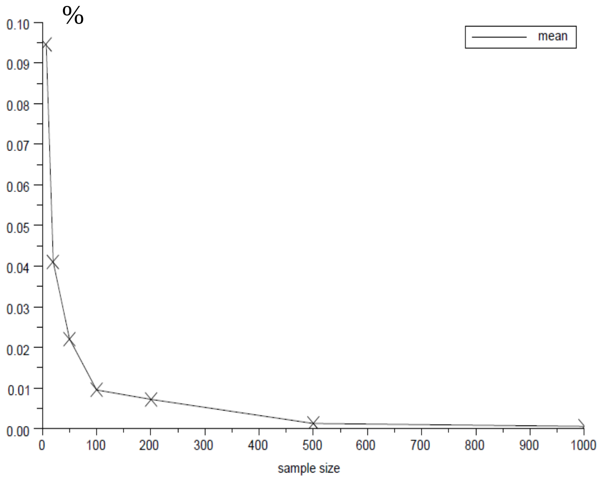


Figure 4: Error percentage on mean average estimates as a function of the sample size.

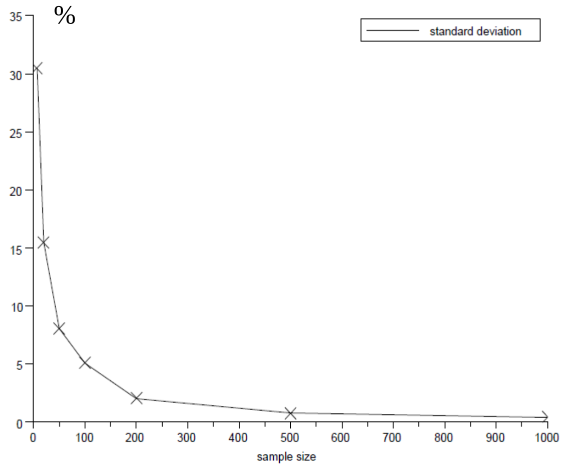


Figure 5: Error percentage on standard deviation average estimates as a function of the sample size.

Figures 4 & 5 show that the StairCase method is converging indeed; the error trends toward zero as the sample size increases, both for the mean and the standard deviation of the fatigue strength.

For very small samples, the method gives acceptable results for mean estimates only. For a sample size of 7, we note an error on the average of the mean estimate lower than 0.1%, whereas the maximum error for the 1000 simulations is about 10% (figure 6, left). Considering the standard deviation estimation, we can see a significant error up to 50% on the standard deviation average, as shown on figure 6 right.

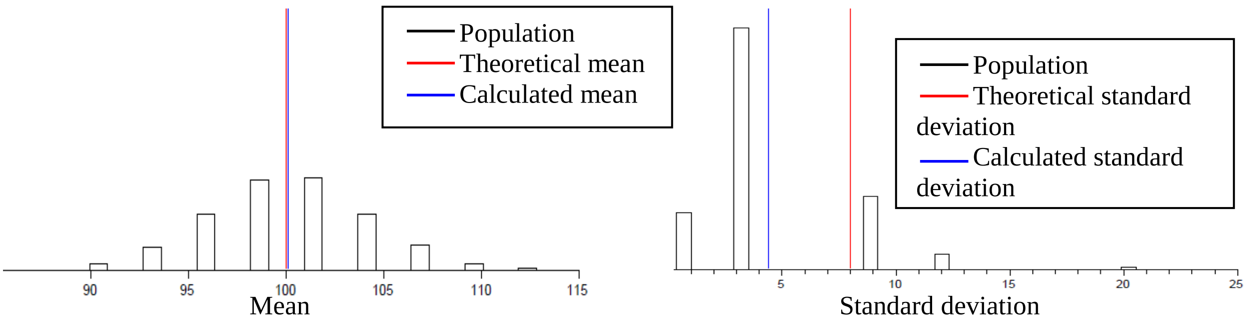


Figure 6: Mean and Std deviation distributions of 1000 runs of a 7 sample size StairCase simulation.

Thus, for small samples, the most common situation in industry, we recover some limits of the D&M application on SC results:

1. the true mean value is never found as a result, even if the result population histogram is centered on the theoretical value;
2. the D&M method applied to a small sample only give sparse values;
3. a more reliable estimator is supposed to be used for the standard deviation.

3 METHOD BASED ON ACCELERATED LIFE TESTING MODEL

3.1 Accelerated Life Testing (ALT) Model

Accelerated Life Testing Models [10, 11] are based on a transfer function and lead to obtain the reliability curve from test results at stresses higher than the usual stress (see figure 7).

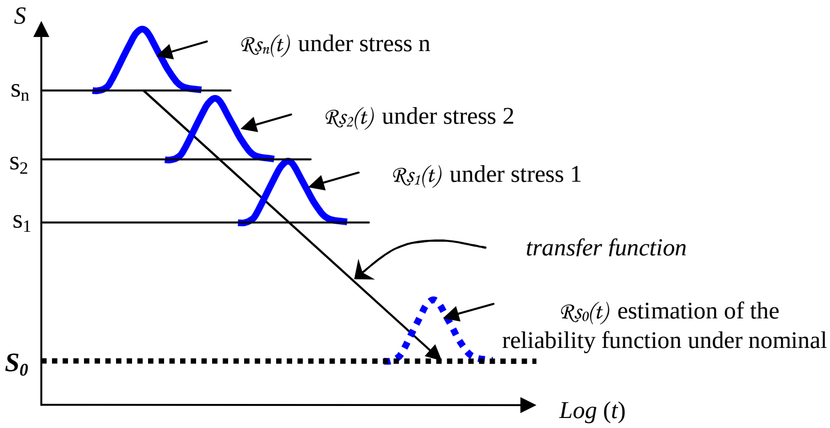


Figure 7: ALT testing principle.

We suppose that we have stresses depending on time:

$$s = s(t)\text{, with } t \geq 0 \text{ and } s \in [0,\infty[\qquad (7)$$

The lifespan $T_{s(.)}$ of a part under constant stress s(.) is a stochastic variable whose reliability function is

$$R_{s(.)}(t) = prob(T_{s(.)} > t), \qquad t \geq 0 \qquad (8)$$

Let R_{s0} be the reliability under usual stress: $s_0 \in \mathcal{E}_0 \subset \mathcal{E}$ ($\mathcal{E}$, a set of constant stress over time), and $R_{s0}{}^{-1}$ its inverse function. The transfer function is defined by:

$$f : [0,+\infty[\times \mathcal{E} \rightarrow [0,+\infty[\qquad (t,s(.)) \rightarrow f(t,s(.)) = (R_{s0}^{-1} \circ R_{s0})(t) \qquad (9)$$

and permits to evaluate the reliability under usual stresses experimentally unavailable, from the reliability under higher stresses.

The standard ALT model is defined over $\mathcal{E}$ if there is a function such as for every $s(.) \in \mathcal{E}$:

$$\frac{d}{dt} f_{s(.)}(t) = r[s(t)] \qquad (10)$$

The function *r* defines a degradation rate. The ALT model states that the rate of resource used at time *t* depends only on the value of the applied stress at time *t*. The equation (10) involves:

$$R_{s(.)}(t) = R_{s_0}\left(\int_0^t r[s(\tau)]d\tau\right) \qquad (11)$$

A special case is $s(\tau) \equiv s = const$, for which :

$$R_s(t) = R_{s_0}(r(s) \cdot t) \qquad (12)$$

So, the stress affects only the scale parameter. Note that $r(s_0)$=1. If the function *r* is completely unknown, the reliability function R_{s0} cannot be estimated. Therefore, the function *r* is chosen to belong to particular classes of function. In survival analysis, the log-linear models defining the ALT models are frequently used as regression models [12].

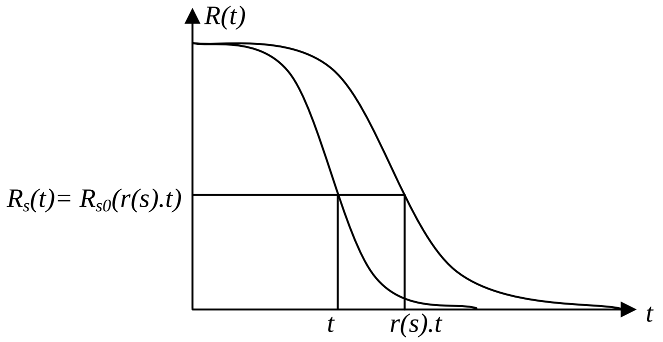


Figure 8: Definition of time transfer regression model r(s).

The log-linear models specify the effect of the covariates as a multiplicative factor for the hazard rate, a scale change, or a location shift for the reliability function, respectively. So the generalization of the model becomes:

$$R_{s(.)}(t) = R_{s_0}\left(\int_0^t e^{\beta^T z(\tau)} d\tau\right) \qquad \text{or} \qquad R_s(t) = R_{s_0}\left(e^{\beta^T z}\right) \tag{13,14}$$

with $\beta = (\beta_0, .., \beta_m)^T$ a parameters vector, $z = (\varphi_0(s), .., \varphi_m(s))^T$ with the functions φ_i based on the different stresses and with the first component z_0 equal to 1. Several models, as Arrhenius, inverse power, generalized Eyring and so on, can be obtained as a particular case of this general form [12].

3.2 Application of ALT Model on StairCase results

The StairCase method is extensively applied, thanks to its independence from physical parameters and because of it provides reliable results using few parts, thus involving low testing time. Nevertheless, as described before (§.2), the D&M estimation of StairCase results does not permit to obtain a reliable estimation of the scatter parameter for the fatigue strength, which is an essential feature for the overall failure risk management. Moreover, we even search for a more reliable mean estimation.

In the sequel, we want to overcame these drawbacks by determining the distribution parameters of the lifespan at a specific usual stress (e.g. the value of the fatigue strength) using an ALT Model.

Considering the ALT application to the fatigue phenomena, the reliability function $R_{s0}(n)$, with n the number of cycles, is defined by a Log-normal distribution:

$$R_{s0}(n) = 1 - \phi\left(\log\left(\left(\frac{n}{\eta}\right)^{\upsilon}\right)\right), \quad (\eta, \upsilon > 0) \tag{15}$$

with ϕ the cumulative distribution function of the standard normal distribution, η and υ the distribution parameters.

So, for a constant stress s, the reliability law defined by the equation (14) is:

$$R_s(n) = 1 - \phi\left(\log\left(\left(\frac{r(s)\cdot n}{\eta}\right)^{\upsilon}\right)\right) = 1 - \phi\left(\log\left(\left(\frac{e^{\beta^T z}\cdot n}{\eta}\right)^{\upsilon}\right)\right) \tag{16,17}$$

$$R_s(n) = 1 - \phi\left(\log\left(\frac{\log(n) - \gamma^T z}{\sigma}\right)\right) \tag{18}$$

where $\gamma = (\gamma_0, \cdots, \gamma_m)$, $\gamma_0 = \log(\eta) - \beta_0$, $\gamma_i = -\beta_i$ and $\sigma = 1/\nu$

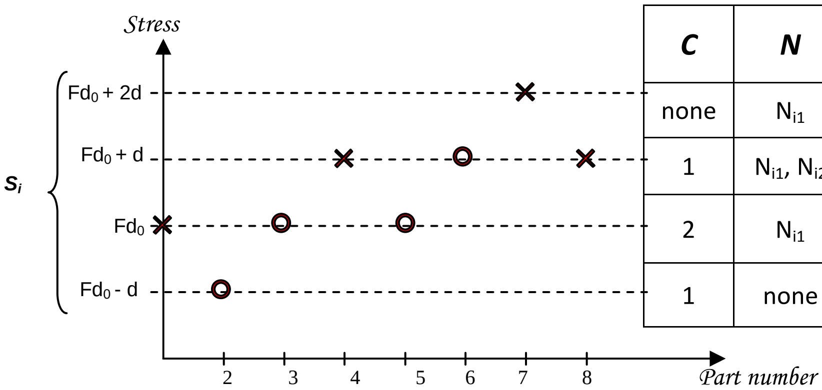
Considering the magnitude of a cyclic stress as the only factor, the Basquin law and the Miner's rule lead to write the reliability function at a constant stress. In fact the Basquin law is chosen to describe the Wöhler curve in high cycle fatigue, by the following relation:

$$N \cdot S^b = A, \text{ or } \log(S) = \frac{1}{b}(\log(A) - \log(N)) \tag{19}$$

According to *§.2.1*, let us consider a StairCase procedure with a first stress level, Fd_0 and a step stress, d at the end of the procedure we have:

- ***i*** stress levels $\boldsymbol{S_i}$ (= Fd_0, Fd_0+ d, Fd_0 + 2d, Fd_0 – d, …),
- at each level we observe ***j*** tested parts,
- $\boldsymbol{N_{ij}}$ are the numbers of cycles to failure at the $\boldsymbol{S_i}$ level for the part ***j***,
- $\boldsymbol{C_i}$ the number of part censured at C=10^6 cycles at the stress level $\boldsymbol{S_i}$.

Following the example below:



C	N
none	N_{i1}
1	N_{i1}, N_{i2}
2	N_{i1}
1	none

Figure 9: Example of a StairCase simulation results.

From (18) and (19) we can deduce the Likelihood function:

$$L(\gamma,\sigma)=\prod_{1}^{i}\left[\prod_{1}^{j}\left(\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\log(N_{ij})-\gamma^{\mathrm{T}}z^{(i)}}{\sigma}\right)^{2}}\right)\times\left(1-\phi\left(\frac{\log(C)-\gamma^{\mathrm{T}}z^{(i)}}{\sigma}\right)\right)^{C_i}\right] \qquad (19)$$

with $\gamma^{\mathrm{T}}z^{(i)}=\log(A)-b\cdot\log(S_i)$, then $\gamma_0=\log(A)$, et $\gamma_1=b$. Applying the Maximum Likelihood method we determine the $\hat{\gamma}$ and $\hat{\sigma}$ parameters estimation.
Then we will conclude on the reliability function at the usual stress:

$$R_{S_0}(N)=1-\phi\left(\frac{\log(N)-\hat{\gamma}^{\mathrm{T}}z^{(0)}}{\hat{\sigma}}\right) \qquad (20)$$

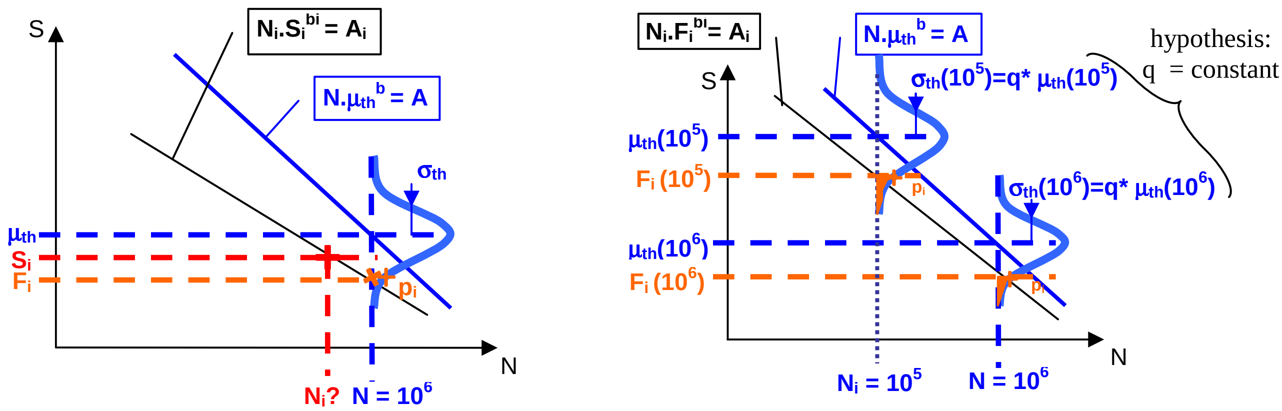
where $z^{(0)}$ corresponds to the nominal (usual) stress condition.

3.3 Numerical Simulation of ALT Model applied to SC results

We now apply ALT method to the results of StairCase simulations as an alternative of Dixon & Mood estimation procedure. These simulations are driven in the same way as described in *§.2.2*. But contrary to D&M estimation, the ALT model needs for the numbers of cycles to failure of each part, or the censured number of cycles, in order to estimate the strength.

The simulating method follows.

At a StairCase stress level $\mathbf{S_i}$ we set a random probability of failure $\mathbf{p_i}$, according the theoretical fatigue strength normal distribution $N(\mu_{th},\sigma_{th})$. We deduce the equivalent stress to failure $\mathbf{F_i}$. As shown on the figure *10.a*, if the equivalent stress is upper than the test stress level ($\mathbf{F_i} > \mathbf{S_i}$) there is no failure at the censured time (10^6) but if $\mathbf{F_i} < \mathbf{S_i}$ we observe a failure at $\mathbf{N_i}$ cycles, with $\mathbf{N_i} < 10^6$.



(a) - way to find of a failure appearance before censure *(b) – iso-probability curves*

Figure 10: Simulating method in order to obtain the number of cycles to failure in a StairCase test plan

The number of cycles de failure $\mathbf{N_i}$ has to be computed. The iso-probability curves are considered different for each part. We determine the parameters for each curve equation (see figure *10.b*) with respect to the PSA hypothesis: the fatigue curve follows a Basquin model and the coefficient of variation of the fatigue Strength is constant. So, we have:

$$\begin{cases} F_i\left(10^5\right)=\Phi^{-1}\left(p_i\middle|\mu_{th}\left(10^5\right),\sigma_{th}\left(10^5\right)\right) \\ F_i\left(10^6\right)=\Phi^{-1}\left(p_i\middle|\mu_{th}\left(10^6\right),\sigma_{th}\left(10^6\right)\right) \end{cases} \tag{21}$$

From Basquin law and the equation (21) it follows:

$$5+b_i\cdot\log\left(F_i\left(10^5\right)\right)=\log\left(A_i\right) \tag{22}$$

$$6+b_i\cdot\log\left(F_i\left(10^6\right)\right)=\log\left(A_i\right) \tag{23}$$

From (22) and (23) we can deduce the parameters of the curve:

$$b_i=-\frac{1}{\log\left(\dfrac{S_i\left(10^6\right)}{S_i\left(10^5\right)}\right)}\quad ;\quad A_i=10^{\frac{11+b_i\cdot\log\left(S_i\left(10^5\right)\cdot S_i\left(10^6\right)\right)}{2}} \tag{24,25}$$

and then calculate the number of cycles to failure for a part at a StairCase stress level $\mathbf{S_i}$:

$$N_i=\frac{A_i}{S_i^{\,b_i}} \tag{26}$$

The same numerical results as used in the Dixon & Mood study are used here, in order to assess the efficiency of the ALT method. So, we consider the results of StairCase simulations driven in "ideal" conditions. The test conditions to apply ALT model are given in Table 1:

Usual value (Normal distribution) $\mathcal{N}(\mathbf{S_0;d})$	Median of the number of cycles to failure at S_0 $(\mathbf{N_0})$	Standard deviation of log-number of cycles at S_0 $(\boldsymbol{\sigma})$	Basquin slope **(b)**	Basquin parameter **(A)** $\left(A=N_0\cdot S_0^{\,b}\right)$	Number of cycles to failure for the $\boldsymbol{j}^{th}$ part at an ***i*** level	Number of censured parts at an ***i*** level
$\mathcal{N}(100;8)$	10^6	0,3	8	10^{22}	$\mathbf{N_{ij}}$	$\mathbf{C_i}$

Table 1: Parameters and conditions applied for the Maximum Likelihood optimisation.

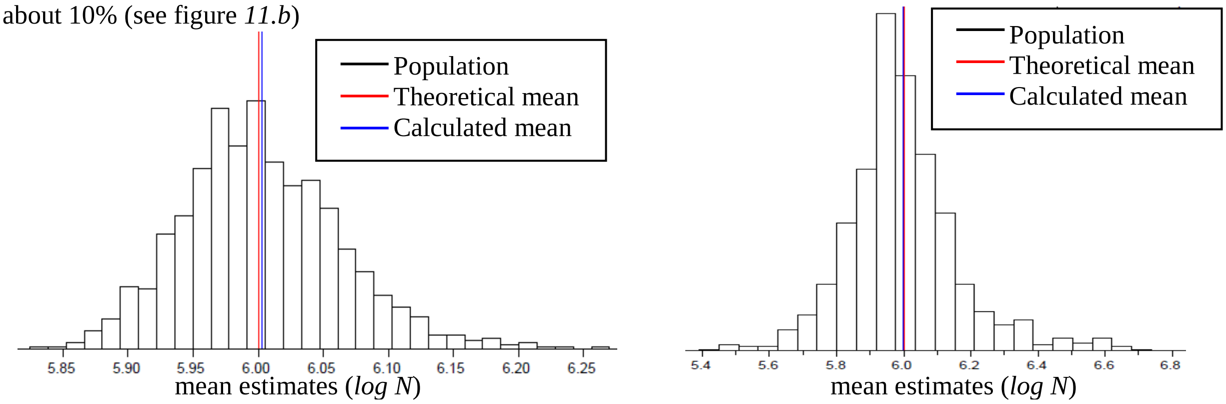
We analyze the data as described in *§.3.2* by Maximum Likelihood Optimization to find the parameters estimators $\hat{\gamma}$ and $\hat{\sigma}$ (*equation 19*), which give the parameters' estimations of the log-number of cycles to failure distribution $\hat{\mu}$ and $\hat{\sigma}$ at S_0, determining:

$$\hat{\mu}=\log\left(\hat{A}\right)-\hat{b}\cdot\log\left(S_0\right),\ \text{with}\ \log\left(\hat{A}\right)=\hat{\gamma}_0\ \text{and}\ \hat{b}=\hat{\gamma}_1. \tag{27}$$

This simulation and calculation are computed several times, so that we can observe the distribution of each parameter (i.e. $\hat{\mu}$ and $\hat{\sigma}$).

Mean estimation

For a large sample size (i.e. 500 parts) we observe an error on the mean estimation of the log-cycles of about 0.05% and a maximum error of 4% (see figure *11.a*). This represent an average error of 5 000 cycles on the mean time to failure at the usual stress, and a maximum error of 100 000 cycles. The error on the mean estimates of a 7 parts sample is not higher, about 0.02% for the average of the mean, and a maximum error of about 10% (see figure *11.b*)

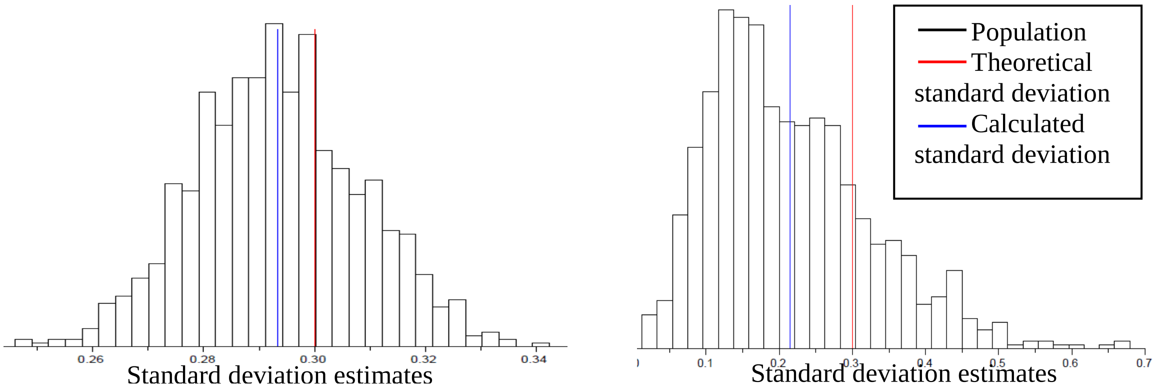


(a) – 500 sample size mean error. *(b) – 7 sample size mean error.*

Figure 11: Mean log-cycles distribution at the usual stress level for: (a) 200, (b) 7 sample size.

Standard deviation estimation:

For a large sample size we observe an error average on the standard deviation less than 2% and of about 13% for the maximum error made on standard deviation estimation (see figure *12.a*). Whereas for small samples the average error is about 25 % and the estimation could lead for some cases at an extreme error of 100%.



(a) – 500 sample size standard deviation error. *(b) – 7 sample size standard deviation error.*

Figure 12: Std deviation distribution of log-cycles at the usual stress level for: (a) 200, (b) 7 sample size.

4 CONCLUSIONS AND PERSPECTIVES

The approach based on ALT Model give results more robust than Dixon & Mood applied to a StairCase test plan under "ideal" conditions. In fact, the estimation of the cycles to failure distribution at a usual stress seems to be more reliable than the estimation of the fatigue strength at a given number of cycles. Moreover, ALT leads to a better estimation of the scatter than Dixon & Mood estimation. These assessments are supported by the following comparison table:

		Maximum error on estimation		Average error on estimation	
Estimation	Sample size	ALT model estimation error	Dixon & Mood estimation error	ALT model estimation error	Dixon & Mood estimation error
Mean	7	**10 %**	**10 %**	**0,02 %**	**0,03 %**
	500	**3 %**	**2 %**	**0,05 %**	**0,01 %**
Standard deviation	7	**100 %**	**More than 150 %**	**25 %**	**50 %**
	500	**13 %**	**30%**	**2 %**	**2 %**

Table 2: Comparison of the error made by the ALT and D&M estimation methods.

Please note that the analysis should be completed by a comparison of the risk engaged by each estimation method. This will be based on a new definition of the problem: in fact, based on the stress-strength method, we are supposed to define the stress distribution also in terms of number of cycles.

Moreover, complementary analyses are expected on the ALT application in order to assess its relevancy; in fact, to observe any bias of the method it is important to check the method results in not "ideal" conditions, i.e. offset procedure conditions, that is:

1. the starting stress level, Fd_0 is different from the true average, μ
2. the step stress value, d is different from the true standard deviation, σ

Finally, the ALT application is supposed to be extended to other testing methods (e.g. Locati, StairCase-Locati). Afterwards, if the ALT analysis confirms its efficiency and robustness, an optimized test plan should be built up (e.g. constant stress test plan, step stress test plan).

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